Impedance Matching for Complex Loads Through Nonuniform Transmission Lines

Gaobiao Xiao and Ken'ichiro Yashiro, *Member, IEEE*

***Abstract*—A numerical method for designing nonuniform transmission lines (NTLs) to match complex loads is presented. This method is based on solving an inverse problem derived from the telegrapher's equation. The matched NTLs are expected to have bandpass characteristics covering the sampling frequency points. A numerical algorithm is provided and verified by examples.**

***Index Terms*—Impedance matching, inverse problem, nonuniform transmission line.**

I. Introduction

IMPEDANCE matching for complex loads is often encountered in microwave engineering. In addition to conventional matching networks, nonuniform transmission lines (NTLs) have also been applied for them. Parabolic tapered transmission lines (PTLs) are investigated by several authors, and a family of equivalent circuits based on Kuroda's identity are available [1]–[3]. They have been used to match lumped series *RC* load, parallel *RL* load, or Brune type sections, while extra reactance elements are usually required to cancel the imaginary part of the transformed impedance. Other NTL synthesis techniques [4]–[10] are, in general, not suitable for handling impedance-matching problems for complex loads.

The authors have proposed a numerical method to synthesize NTLs in [11], and have successfully applied it to designs of filter. However, since both the amplitude and phase characteristics of S-parameter $S_{11}(j\omega)$ are required in the method, it is not convenient for synthesizing tapered NTLs for matching complex loads. This paper extends the method, and make it possible to design tapered NTLs for matching complex loads from $|S_{11}(j\omega)|$ alone. No extra reactance element is needed. Design theory and formulas are discussed in Section II, while an algorithm will be described in Section III. Design examples for lumped series *RC* will also be illustrated. The present method may seem to be a little complicated, but is not difficult to implement in a personal computer.

II. Design Theory

Let the current and voltage in a lossless NTL be denoted by $I(z,\omega)$ and $V(z,\omega)$. They must satisfy the telegrapher's equations $dV/dz = -j\omega L(z)I, dI/dz = -j\omega C(z)V$. Define electrical position as [12], [13]

$$x(z) = f_c \int_0^z \sqrt{L(s)C(s)}\, ds \tag{1}$$

and the characteristic impedance of the line as $Z_0(x) = \sqrt{L[z(x)]/C[z(x)]}$. x is normalized in interval $(0,1)$, which implies that the length of the NTL is chosen as $\ell = \lambda_c$, λ_c is the guided wavelength in NTLs at normalizing frequency f_c.[1]

The telegrapher's equation can be deduced to the following Sturm–Liouville's equation:

$$\phi''(x) + [\lambda - q(x)]\phi(x) = 0 \tag{2}$$

and

$$\frac{d\phi(x)}{dx} - k(x)\phi(x) + j\omega\psi(x) = 0 \tag{3}$$

where $\phi(x,\omega) = V(x,\omega)/\sqrt{Z_0(x)}$, $\psi(x,\omega) = I(x,\omega)\sqrt{Z_0(x)}$, and $\lambda = \omega^2$. The potential function $q(x)$ and parameter $k(x)$ (sometimes called the local reflection coefficient) satisfy

$$\left(\frac{1}{\sqrt{Z_0(x)}}\right)'' - q(x)\frac{1}{\sqrt{Z_0(x)}} = 0 \tag{4}$$

$$\left(\frac{1}{\sqrt{Z_0(x)}}\right)' - k(x)\frac{1}{\sqrt{Z_0(x)}} = 0. \tag{5}$$

The incident and reflected waves are defined by $2h_g a_1 = V(0,j\omega) + Z_g I(0,j\omega)$, $2h_g^* b_1 = V(0,j\omega) - Z_g^* I(0,j\omega)$ and $2h_\ell a_2 = V(1,j\omega) - Z_\ell I(1,j\omega)$, $2h_\ell^* b_2 = V(1,j\omega) + Z_\ell^* I(1,j\omega)$, where $2h_g h_g^* = Z_g + Z_g^*, 2h_\ell h_\ell^* = Z_\ell + Z_\ell^*$. Under these definitions, the reflection coefficient for a matched NTL is $|\Gamma(j\omega)| = |S_{11}(j\omega)|$.

On the other hand, we denote the two linearly independent solutions of (2) as $y_1(x,\lambda)$ and $y_2(x,\lambda)$, where $y_1(0,\lambda) = 1, y_1'(0,\lambda) = 0$ and $y_2(0,\lambda) = 0, y_2'(0,\lambda) = 1$. By a similar analysis method provided in [11], $S_{11}(j\omega)$ and $S_{21}(j\omega)$ can be expressed in terms of $y_1(1,\lambda), y_2(1,\lambda), y_1'(1,\lambda), y_2'(1,\lambda)$ and four parameters of $k(0), k(1), Z_0(0)$ and $Z_0(1)$ as

$$S_{11}(j\omega) = \frac{N(j\omega)}{D(j\omega)} \tag{6}$$

$$S_{21}(j\omega) = \frac{2j\omega}{D(j\omega)} \tag{7}$$

Manuscript received February 10, 2001.

The authors are with the Department of Electronics and Mechanical Engineering, Chiba University, Chiba 263-8522, Japan.

Publisher Item Identifier S 0018-9480(02)05199-2.

[1] This flexible choice is very convenient. The practical length of the NTL can be obtained when the characteristic impedance profile is determined.

where

$$D(j\omega) = \frac{1}{\sqrt{Z_0(0)Z_0(1)}}\frac{Z_\ell Z_g}{h_\ell h_g} \times \left\{ \left[\left(G_1(\lambda) - k(1)G_2(\lambda)\right) - \frac{Z_0(0)Z_0(1)}{Z_\ell Z_g}\omega^2 y_2(1,\lambda) \right] + j\omega \left[\frac{Z_0(1)}{Z_\ell}G_2(\lambda) - \frac{Z_0(0)}{Z_g}\left(k(1)y_2(1,\lambda) - y_2'(1,\lambda)\right) \right] \right\} \quad (8)$$

$$N(j\omega) = \frac{1}{\sqrt{Z_0(0)Z_0(1)}}\frac{Z_\ell Z_g^*}{h_\ell h_g^*} \times \left\{ \left[-\left(G_1(\lambda) - k(1)G_2(\lambda)\right) - \frac{Z_0(0)Z_0(1)}{Z_\ell Z_g^*}\omega^2 y_2(1,\lambda) \right] - j\omega \left[\frac{Z_0(1)}{Z_\ell}G_2(\lambda) + \frac{Z_0(0)}{Z_g^*}\left(k(1)y_2(1,\lambda) - y_2'(1,\lambda)\right) \right] \right\} \quad (9)$$

and

$$G_1(\lambda) = y_1'(1,\lambda) + k(0)y_2'(1,\lambda) \quad (10)$$

$$G_2(\lambda) = y_1(1,\lambda) + k(0)y_2(1,\lambda). \quad (11)$$

The main idea of the present synthesis method is: 1) using the above formulas to approximate a requisite matching characteristics for given load and source impedances Z_ℓ and Z_g by selecting proper functions of $y_1(1,\lambda), y_2(1,\lambda), y_1'(1,\lambda), y_2'(1,\lambda)$, and the four parameters of $k(0), k(1), Z_0(0)$ and $Z_0(1)$ and 2) constructing the potential function $q(x)$ and the characteristic impedance profile $Z_0(x)$ of an NTL from those functions and parameters.

In order to describe our algorithm clearly, we are going to quote some important properties involved at first.

1) All four functions of $y_1(1,\lambda), y_1'(1,\lambda), y_2(1,\lambda)$, and $y_2'(1,\lambda)$ are entire functions of order $1/2$ (type 1) and so are their linear combinations [14], [15]. They can be determined from their zeros and their asymptotic behaviors uniquely. Especially if $\int_0^1 q(s)ds = 0$, the four entire functions can be expressed in the following easy calculating forms (see Appendix A):

$$y_1(1,\lambda) = \cos\sqrt{\lambda}\sum_1^M \frac{\alpha_i - \lambda}{P_i - \lambda} \quad (12)$$

$$y_1'(1,\lambda) = (\beta_0 - \lambda)\frac{\sin\sqrt{\lambda}}{\sqrt{\lambda}}\sum_1^M \frac{\beta_i - \lambda}{Q_i - \lambda} \quad (13)$$

$$y_2(1,\lambda) = \frac{\sin\sqrt{\lambda}}{\sqrt{\lambda}}\sum_1^M \frac{\gamma_i - \lambda}{Q_i - \lambda} \quad (14)$$

$$y_2'(1,\lambda) = \cos\sqrt{\lambda}\sum_1^M \frac{\mu_i - \lambda}{P_i - \lambda} \quad (15)$$

where $\alpha_i, \beta_i, \gamma_i$ and μ_i are zeros of $y_1(1,\lambda)$, $y_1'(1,\lambda), y_2(1,\lambda)$ and $y_2'(1,\lambda)$, respectively. $P_i = [(i-0.5)\pi]^2$, $Q_i = (i\pi)^2$. M is the number of zeros that we want to modify.

2) The Wronskian of $y_1(1,\lambda)$ and $y_2(1,\lambda)$ satisfy

$$W(y_1, y_2) = y_1(1,\lambda)y_2'(1,\lambda) - y_1'(1,\lambda)y_2(1,\lambda) = 1. \quad (16)$$

We have verified that, in the case of a lossless NTL, the above Wronskian will lead to the well-known relationship

$$|S_{11}(j\omega)|^2 + |S_{21}(j\omega)|^2 = 1. \quad (17)$$

3) The potential function $q(x)$ can be uniquely constructed either from the two sequences of α_i and β_i or from γ_i and μ_i. The two potentials generated in these two ways are in agreement if the above Wronskian relationship is satisfied.

These properties show that a preassigned frequency response can be approximated through adjusting the four zero sequences of $\alpha_i, \beta_i, \gamma_i$, and μ_i, and the four parameters of $k(0), k(1), Z_0(0)$, and $Z_0(1)$, and from those zero sequences a potential function $q(x)$ can be constructed. Recalling that $q(x)$ relates with $Z_0(x)$, these formulas actually have associated the frequency response of an NTL with its fabrication feature. However, there still remains another problem to cope with. By virtue of (5), we will find that $k(0) = -Z_0'(0)/[2Z_0(0)]$ and $k(1) = -Z_0'(1)/[2Z_0(1)]$. Thus, there exist four boundary data for the second-order equation (4). In order that there exist nontrivial solutions when solving $Z_0(x)$ from this equation, the four parameters $Z_0(0), Z_0(1), k(0)$, and $k(1)$ must satisfy the following additional conditions (see Appendix B):

$$y_1(1,0) + k(0)y_2(1,0) = \sqrt{\frac{Z_0(0)}{Z_0(1)}} \quad (18)$$

$$y_1'(1,0) + k(0)y_2'(1,0) = k(1)\sqrt{\frac{Z_0(0)}{Z_0(1)}}. \quad (19)$$

Obviously, the problem of approximating a requisite matching characteristics may be cast into a constrained nonlinear optimization problem.

In practice, complete matching can be achieved only at some discrete frequency points through a tapered NTL with finite length. Here, we assume to design an NTL to match Z_ℓ and Z_g on the condition that $|S_{11}(j\omega)| = \rho_{si}$ ($\rho_{si} = 0$ for complete matching) at discrete sampling frequencies $\omega_{si} = \sqrt{\lambda_{si}}$ ($i = 1, \ldots, N$). The design may be accomplished in the following two steps.

Step 1) Determine $\alpha_i, \beta_i, \gamma_i, \mu_i$ and $Z_0(0), Z_0(1), k(0), k(1)$ from minimizing

$$E_s = \sum_1^N \left[|S_{11}(j\omega_{si})| - \rho_{si}\right]^2 \quad (20)$$

being subject to conditions (16), (18), and (19). $|S_{11}(j\omega)|$ is calculated from (6).

Condition (16) is inconvenient to apply directly in calculation because it must be satisfied for all values of λ. In the algorithm described below, we only request that this Wronskian relationship be satisfied for all four zero sequences of $\alpha_i, \beta_i, \gamma_i$, and μ_i.[2] Based on this consideration, we defined an error function from (16) as follows:

$$E_w = \sum_1^M \left[y_1'(1,\alpha_i)y_2(1,\alpha_i)+1\right]^2 + \sum_1^M \left[y_1'(1,\mu_i)y_2(1,\mu_i)+1\right]^2 + \sum_1^M \left[y_1(1,\beta_i)y_2'(1,\beta_i)-1\right]^2 + \sum_1^M \left[y_1(1,\gamma_i)y_2'(1,\gamma_i)-1\right]^2 \quad (21)$$

and from conditions (18) and (19), we defined

$$E_z = \left[G_2(0) - \sqrt{\frac{Z_0(0)}{Z_0(1)}}\right]^2 + \left[G_1(0) - k(1)\sqrt{\frac{Z_0(0)}{Z_0(1)}}\right]^2. \quad (22)$$

$\alpha_i, \beta_i, \gamma_i, \mu_i$ and $Z_0(0), Z_0(1), k(0), k(1)$ are then determined from minimizing the following object function:

$$E_r(X) = E_s(X) + c_0E_z(X) + c_1E_w(X) \quad (23)$$

where vector $X = (\alpha_j, \beta_j, \gamma_j, \mu_j, Z_0(0), Z_0(1), k(0), k(1))$. c_0, c_1 are weighting factors. Naturally, there are some limitations on controlling $|S_{11}|$ when an NTL with finite length is used. It is difficult to predict to what extent $|S_{11}|$ can be controlled for arbitrary Z_ℓ and Z_g. In the special case of real Z_ℓ and Z_g, $|S_{11}|$ is also an entire function of order not larger than $1/2$. Thus, $|S_{11}|$ can only be controlled at a discrete frequency sequence with asymptotic behavior like those of the above-mentioned zero sequences. For complex Z_ℓ and Z_g, the frequency range that can be controlled tends to be much narrower. For impedances with large imaginary parts, longer NTLs may be required to reach satisfactory matching.

Step 2) Construct $q(x)$ from solving the inverse problem of (2), using either α_i, β_i or γ_i and μ_i.

This has already been widely investigated, and a numerical algorithm is available [16]. $Z_0(x)$ can be calculated from $q(x)$. To calculate the practical dimensions of the NTL, we have to represent z by x. For microstrip lines, we may first calculate the linewidth $W(x)$ and effective dielectric constant $\epsilon_{\text{eff}}(x)$ from $Z_0(x)$ [17], and then calculate z from

$$z = \frac{c}{f_c}\int_0^x \frac{1}{\sqrt{\epsilon_{\text{eff}}(s)}}\,ds \quad (24)$$

where c is the light velocity in free space. Therefore, the length of the tapered NTL is $\ell = (c/f_c)\int_0^1 (1/\sqrt{\epsilon_{\text{eff}}(s)})\,ds$, and the practical dimensions of the NTL can be determined from $W_p(z(x)) = W(x)$.

[2]We cannot prove that the Wronskian is strictly satisfied for all λ when it is satisfied for the four zero sequences, but the errors stemmed from this simplification can be expected to be very small due to the features of the four unctions involved.

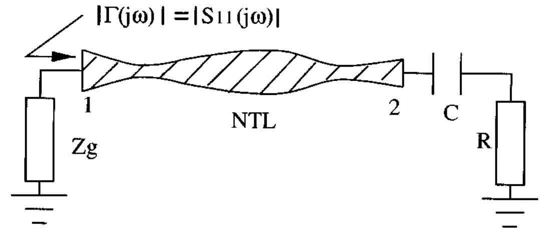


Fig. 1. Matching a lumped series *RC* load through an NTL.

III. Algorithm and Examples

Let $\rho_{si} = 0$. It is readily verified that h_g, h_g^*, h_ℓ and h_ℓ^* are not necessary to calculate in calculating $|S_{11}(j\omega)|$. Initial data may be chosen as $Z_0(0) = \text{Re}(Z_g), Z_0(1) = \text{Re}(Z_\ell), k(0) = k(1) = 0$, and $\alpha_i = \mu_i = P_i, \beta_i = \gamma_i = Q_i$ for $i = 1, 2, \ldots$. $\beta_0 = 0$. The optimization process may be carried out iteratively by adopting

$$X_i^{(n+1)} = X_i^{(n)} - \tau\frac{\partial E_r}{\partial X_i} \quad (25)$$

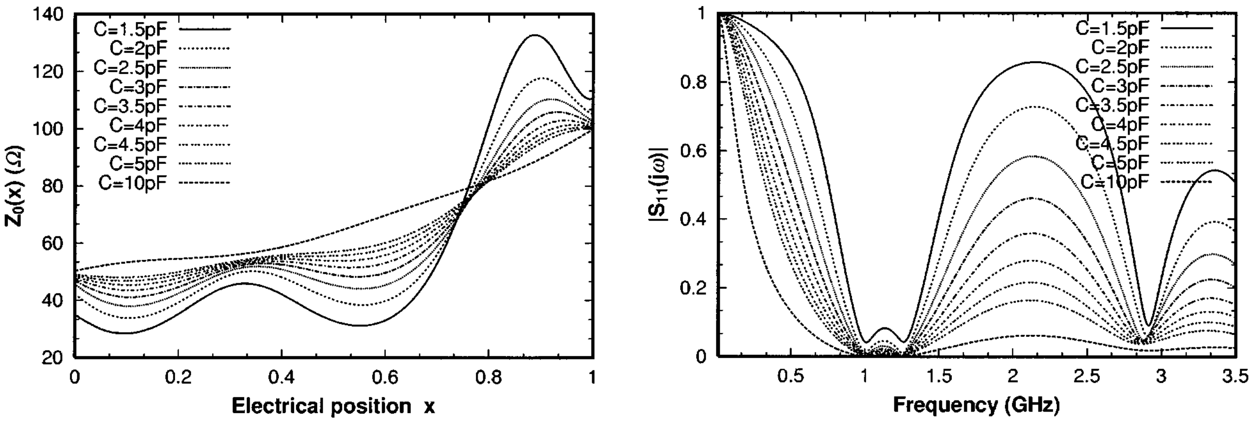
where τ must be sufficiently small so that the process is stable. Generally, τ should use a smaller value for larger M and N.

Without loss of generality, we consider the matching problem in Fig. 1, where $Z_g = 50\,\Omega$, and Z_ℓ is a lumped series *RC* circuit. $R = 100\,\Omega$. Numerical results for three cases are as follows.

Case 1) Normalizing frequency is chosen as $f_c = 2$ GHz. C varies from 1.5 to 10 pF. Sampling points are fixed at $f_{s1} = 1$ GHz and $f_{s2} = 1.25$ GHz. Thus, we have $\omega_{s1} = 2\pi f_{s1}/f_c = \pi$ and $\omega_{s2} = 1.25\pi$. Fig. 2 show the optimized $Z_0(x)$ and the corresponding simulated results of $|S_{11}(j\omega)|$, respectively. Smaller series C causes a larger imaginary part of $Z_\ell|S_{11}(j\omega)|$ in the passband for optimized NTLs tend to become larger. Longer NTLs may be required to achieve more satisfactory matching for smaller C.

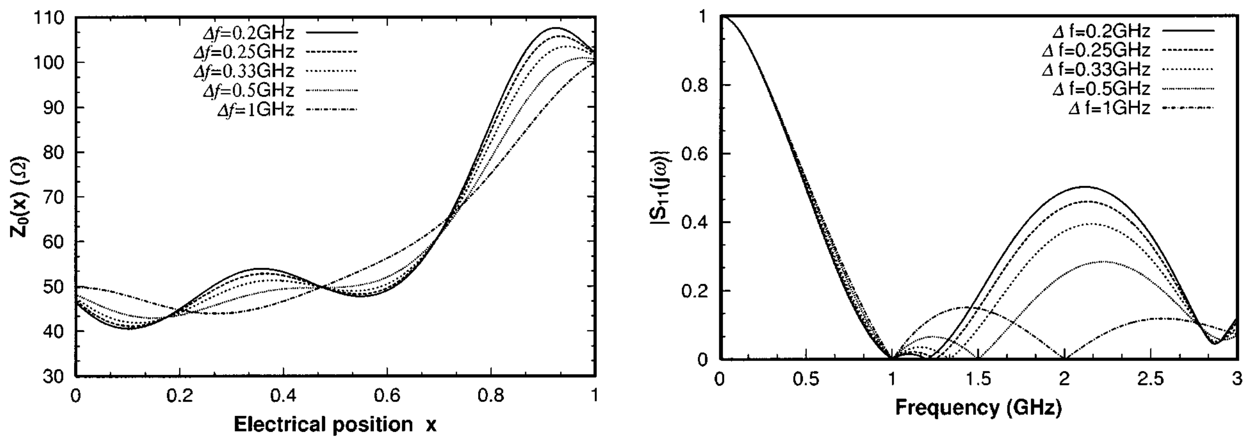
Case 2) $C = 3$ pF, $f_c = 2$ GHz. Two controlling points are also selected, but the sampling spacing varies from 0.2 to 1 GHz. The numerical results are shown in Fig. 3. It can be seen that the ripple between the two sampling points is higher for a larger sampling spacing.

Case 3) In practical applications, in order to obtain an NTL that can be connected to outer lines continuously, we may choose $Z_0(0)$ and $Z_0(1)$ to be fixed and put them out of the optimization process. Though this arrangement may deteriorate the matching characteristics, the side effect can be compensated by using a longer NTL. An example is also provided, where $Z_0(0) = 50\,\Omega$, $Z_0(1) = 100\,\Omega$. $C = 3$ pF. Controlling frequencies are chosen as $f_{si} = [1 + 0.125(i-1)]$ GHz, $i = 1, \ldots, N$. Normalizing frequencies are chosen as $f_c = 1$ GHz. Fig. 4 shows the optimized $Z_0(x)$ and relating simulated $|S_{11}(j\omega)|$. For com-



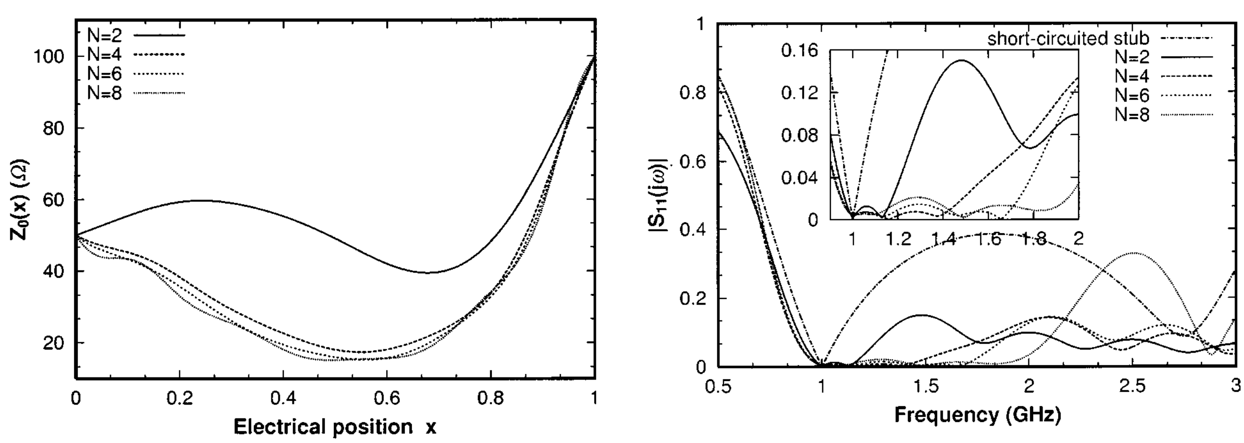
(a) (b)

Fig. 2. Tapered NTLs for case 1: $f_c = 2$ GHz. $Z_g = 50\,\Omega$. $R = 100\,\Omega$. $C = 1.5$ pF $\sim$ 10 pF. Two sampling points are fixed at $f_{s1} = 1$ GHz, $f_{s2} = 1.25$ GHz. $M = 20$. (a) Optimized $Z_0(x)$. (b) Simulated $|S_{11}(j\omega)|$.



(a) (b)

Fig. 3. Tapered NTLs for case 2: $f_c = 2$ GHz. $Z_g = 50\,\Omega$. $R = 100\,\Omega$. $C = 3$ pF. Two sampling points are adopted with $\Delta f = 0.2, 0.25, 0.33, 0.5, 1$ GHz. $M = 20$. (a) Optimized $Z_0(x)$. (b) Simulated $|S_{11}(j\omega)|$.



(a) (b)

Fig. 4. Tapered NTLs for case 3: $f_c = 1$ GHz. $Z_g = 50\,\Omega$. $R = 100\,\Omega$, $C = 3$ pF. $f_{si} = [1 + 0.125(i-1)]$ GHz, $i = 1, \ldots, N$. $M = 20$ for $N \leq 4$, $M = 40$ for $N > 4$. (a) Optimized $Z_0(x)$. (b) Simulated $|S_{11}(j\omega)|$.

parison, in Fig. 4(b), a simulated result of a conventional short-circuited stub matching network for the same Z_g and Z_ℓ is also presented. Apparently, wider matching band can be provided through an NTL.

IV. Conclusion

The numerical method described in this paper can be used to design tapered NTLs for matching complex loads. Numerical examples presented not only validate this method, but also suggest some ways to control the frequency range of matching and the passband ripple levels of a tapered NTL.

Appendix A

We take (12) as an example.

It is already known that $y_1(1,\lambda)$ can be expressed as [14]

$$y_1(1,\lambda) = C\prod_{i=1}^{\infty}\left(1-\frac{\lambda}{\alpha_i}\right) \tag{26}$$

where C is a constant, and α_i are zeroes of $y_1(1,\lambda)$. From the estimate of $y_1(1,\lambda) = \cos(\sqrt{\lambda})[1+o(1)]$ as $\lambda \to -\infty$ [15], we have

$$\lim_{\lambda\to-\infty}\frac{y_1(1,\lambda)}{\cos(\sqrt{\lambda})} = 1. \tag{27}$$

Using $\cos(\sqrt{\lambda}) = \prod_{i=1}^{\infty}(1-(\lambda/P_i))$, the left-hand side of (27) can be further written as

$$\lim_{\lambda\to-\infty}\frac{C\prod_{i=1}^{\infty}\left(1-\frac{\lambda}{\alpha_i}\right)}{\prod_{i=1}^{\infty}\left(1-\frac{\lambda}{P_i}\right)} = \lim_{\lambda\to-\infty} C\prod_{i=1}^{\infty}\frac{\left(1-\frac{\lambda}{\alpha_i}\right)}{\left(1-\frac{\lambda}{P_i}\right)}. \tag{28}$$

When $\int_0^1 q(s)ds = 0$, we have $\alpha_i = P_i + O(i^{-2})$. It is not difficult to check that the canonical multiplication at the right-hand side of (28) is uniformly convergent, at least in an interval of $(-\infty, 0)$. This enable us to pass the limit $\lambda \to -\infty$ into the product in (28). Thus,

$$C\prod_{i=1}^{\infty}\left[\lim_{\lambda\to-\infty}\frac{\left(1-\frac{\lambda}{\alpha_i}\right)}{\left(1-\frac{\lambda}{P_i}\right)}\right] = C\prod_{i=1}^{\infty}\frac{P_i}{\alpha_i} = 1. \tag{29}$$

Therefore, $C = \prod_{i=1}^{\infty}(\alpha_i/P_i)$. Hence,

$$y_1(1,\lambda) = \prod_{i=1}^{\infty}\frac{\alpha_i-\lambda}{P_i}. \tag{30}$$

Furthermore, if we assume that $\alpha_i = P_i$ for $i > M$, then

$$\begin{aligned} y_1(1,\lambda) &= \prod_{i=1}^{M}\frac{\alpha_i-\lambda}{P_i}\prod_{i=M+1}^{\infty}\frac{P_i-\lambda}{P_i} \\ &= \prod_{i=1}^{M}\frac{\alpha_i-\lambda}{P_i-\lambda}\prod_{i=1}^{\infty}\frac{P_i-\lambda}{P_i} \\ &= \cos(\sqrt{\lambda})\prod_{i=1}^{M}\frac{\alpha_i-\lambda}{P_i-\lambda}. \end{aligned} \tag{31}$$

Appendix B

Clearly, (4) is a special case of (2) for $\lambda = 0$. Thus, we can write

$$\frac{1}{\sqrt{Z_0(x)}} = c_1 y_1(x,0) + c_2 y_2(x,0). \tag{32}$$

From the boundary conditions at $x = 0$, we have $c_1 = 1/\sqrt{Z_0(0)}$ and $c_2 = c_1 k(0)$. Hence,

$$\frac{1}{\sqrt{Z_0(x)}} = \frac{1}{\sqrt{Z_0(0)}}\left[y_1(x,0) + k(0)y_2(x,0)\right] \tag{33}$$

and at $x = 1$, the following relations must be observed:

$$y_1(1,0) + k(0)y_2(1,0) = \sqrt{\frac{Z_0(0)}{Z_0(1)}} \tag{34}$$

$$y_1'(1,0) + k(0)y_2'(1,0) = k(1)\sqrt{\frac{Z_0(0)}{Z_0(1)}}. \tag{35}$$

References

[1] K. Kobayashi, Y. Nemoto, and R. Sato, "Kuroda's identity for mixed lumped and distributed circuits and their application to nonuniform transmission lines," *IEEE Microwave Theory Tech.*, vol. MTT-29, pp. 81–86, Feb. 1981.

[2] I. Endo, Y. Nemoto, and R. Sato, "Impedance transformation and matching for lumped complex load with nonuniform transmission lines," *IEEE Microwave Theory Tech.*, vol. MTT-33, pp. 4–9, Jan. 1985.

[3] R. Sato, Y. Ohba, Y. Nemoto, and K. Kobayashi, "Equivalent transformations for the mixed lumped Brune-type section and uniform transmission line," *IEEE Microwave Theory Tech.*, vol. 38, pp. 1114–1119, Aug. 1990.

[4] S. I. Orlov, "Concerning the theory of nonuniform transmission lines" (in Russian), *J. Tech. Phys. USSR*, vol. 26, pp. 2361–2371, 1956.

[5] F. Huang, "Quasitransversal synthesis of microwave chirped filter," *Electron. Lett.*, vol. 28, pp. 1062–1064, May 1992.

[6] G. H. Song and S. Y. Shin, "Design of corrugated waveguide filters by the Gelfand–Levitan–Marchenko inverse scattering method," *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 2, no. 11, pp. 1905–1915, 1985.

[7] P. P. Roberts and G. E. Town, "Design of microwave filters by inverse scattering," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 739–743, Apr. 1995.

[8] M. L. Roy, A. Pèrennec, S. Toutain, and L. C. Calvez, "The continuously varying transmission-line technique-application to filter design," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 1680–1687, Sept. 1999.

[9] G. B. Xiao, K. Yashiro, N. Guan, and S. Ohkawa, "An effective method for designing nonuniformly coupled transmission line filters," *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1027–1031, June 2001.

[10] J. P. Mahon and R. S. Elliott, "Tapered transmission lines with a controlled ripple response," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1415–1420, Oct. 1990.

[11] G. B. Xiao, K. Yashiro, N. Guan, and S. Ohkawa, "A new numerical method for synthesis of arbitrarily terminated lossless nonuniform transmission lines," *IEEE Microwave Theory Tech.*, vol. 49, pp. 369–376, Feb. 2001.

[12] D. C. Youla, "Analysis and synthesis of arbitrarily terminated lossless nonuniform lines," *IEEE Trans. Circuit Theory*, vol. CT-11, pp. 363–372, Sept. 1964.

[13] M. W. Wohlers, "A realizability theory for smooth lossless transmission lines," *IEEE Trans. Circuit Theory*, vol. CT-13, pp. 356–363, Sept. 1966.

[14] B. M. Levitan and M. G. Gasymov, "Determination of a differential equation by two of its spectra," *Russ. Math. Surveys*, vol. 19, pp. 1–63, 1964.

[15] E. Trubowitz, "The inverse problem for periodic potentials," *Commun. Pure Appl. Math.*, vol. 30, pp. 321–337, 1977.

[16] W. Rundell and P. E. Sacks, "Construction techniques for classical inverse Sturm–Liouville problems," *Math. Comput.*, vol. 58, pp. 161–183, Jan. 1992.

[17] M. V. Schneider, "Microstrip lines for microwave integrated circuits," *Bell Syst. Tech. J.*, pp. 1421–1444, May–June 1969.

Gaobiao Xiao was born in China, in 1965. He received the B.S. degree from the Huazhong University of Science and Technology, Wuhan, China, in 1988, and the M.S. degree from the Changsha Institute of Technology, Changsha, China, in 1991, and is currently working toward the D.E. degree in electrical and electronics engineering from Chiba University, Chiba, Japan.

From 1991 to 1997, he was a Research Associate with the Department of Electrical and Electronics Engineering, Hunan University. His main research interests are microwave filters, fiber-optic filters, inverse scattering problems, circuits analysis, and numerical methods in the electromagnetic field.

Ken'ichiro Yashiro (M'83) received the B.E. and M.E. degrees in electronics engineering from Chiba University, Chiba, Japan, in 1974 and 1976, respectively, and the D.E. degree from the Tokyo Institute of Technology, Tokyo, Japan, in 1979.

In 1979, he joined the Department of Electronics and Mechanical Engineering, Chiba University, as a Research Associate, became an Associate Professor in 1987, and a Full Professor in 1998. His research interests include magnetostatic wave devices, magnetic solitons, electromagnetic wave scattering, and inverse scattering problems.